Math 10A with Professor Stankova
Quiz 7; Wednesday, 10/11/2017
Section #106; Time: 10 AM
GSI name: Roy Zhao **Name:** _____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. **TRUE** False We can use the addition differentiation law to prove the addition integration law for indefinite integrals ($\int(f+g) = (\int f) + (\int g)$).

2. **TRUE** False Integrating an integrable function $f(u)$ from a to x gives us a specific anti-derivative of f.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (3 points) Write $\int_1^4 \sqrt{1-\sqrt{x}}dx$ as a limit of right endpoint Riemann sums (no need to calculate it).

Solution: The interval $[1,4]$ has length 3 so each subinterval will have length $\frac{3}{n}$. These subintervals are $[1, 1+3/n], [1+3/n, 1+6/n], \ldots, [4-3/n, 4]$. Using the right endpoint method, we have

$$\int_1^4 \sqrt{1-\sqrt{x}}dx = \lim_{n\to\infty} R_n$$

$$= \lim_{n\to\infty}\left[\sqrt{1-\sqrt{1+\frac{3}{n}}}\cdot\frac{3}{n} + \sqrt{1-\sqrt{1+\frac{6}{n}}}\cdot\frac{3}{n} + \cdots + \sqrt{1-\sqrt{4}}\cdot\frac{3}{n}\right]$$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{1-\sqrt{1+\frac{3i}{n}}}\cdot\frac{3}{n}.$$

(b) (7 points) Find $\int \sqrt{1-\sqrt{x}}dx$

Solution: We guess $u = \sqrt{1-\sqrt{x}}$ and so $u^2 = 1-\sqrt{x}$ and $x = (u^2-1)^2 = u^4 - 2u^2 + 1$. So we have that $dx = (4u^3 - 4u)du$ and so

$$\int \sqrt{1-\sqrt{x}}dx = \int u(4u^3-4u)du = \int 4u^4 - 4u^2 du.$$

$$= \frac{4u^5}{5} - \frac{4u^3}{3} + C = \frac{4(1-\sqrt{x})^{5/2}}{5} - \frac{4(1-\sqrt{x})^{3/2}}{3} + C.$$